Application of the Integro-Differential Method for Precise Computation of the Magnetic Fields in Transformers.

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Integro-differential method is applied for computation of the magnetic field in the short-circuited transformer. For achieving a high precision of the magnetic field intensity computation a finite element technique was used for the post-processing. The developed procedure ensured a high accuracy of the transformer inductances computation with the constant relative permeability of steel. The results of a numerical modeling were approved by comparing with experimental data.

***Index Terms*—Integral equations, finite element method, transformer, magnetic field, magnetic flux, inductance.**

I. INTRODUCTION

APPLICATION of methods based on solutions of space integral equations is often advantageous for simulating magnetic fields because the problem domain in frames of these formulations is restricted only by the volume filled with the magnetic material [1], [2]. One of such formulation is based on a solution of the integro-differential equation for the scalar magnetic potential [3], [4]:

$$U(\vec{r}) + \frac{1}{4\pi}\int_{V_m}\frac{(\mu_r - 1)\vec{\nabla}U(\vec{r}\,')\cdot(\vec{r}-\vec{r}\,')}{\left|\vec{r}-\vec{r}\,'\right|^3}dV_m = U_c(\vec{r}), \qquad (1)$$

where $U(\vec{r})$ and $U_c(\vec{r})$ are the total scalar potential inside the magnetic material and the potential induced by the external sources. In the case of the closed magnetic circuits appropriate cuts should be introduced to provide the existence and uniqueness of the scalar potential. Here we apply this approach for analysing magnetic fields in a system with a closed magnetic circuit which requires a high precision of the field modelling.

II. PECULARITIES OF A SHORT CIRCUIT OPERATION MODE IN ELECTRIC TRANSFORMERS

To estimate the stability of the power transformers the analysis of their steady short-circuit mode is very important. For solving such problems it is necessary to predict the distribution of the magnetic fluxes in the elements of the ferromagnetic core and coils of the transformer.

In the short-circuit mode the main part of the magnetic fluxes does not pass only through the magnetic circuit. That is why a precise evaluation of the magnetic flux density distributions is necessary simultaneously inside the steel core and in surrounding space. The accuracy of computations may be degraded strongly in such a case because of two main reasons: (a) the relative magnetic permeability of the core is typically exceeding values of several thousand; (b) the magnetic coupling of the transformer windings is very close to one unit, so their currents in the short circuit mode may exceed strongly the normal values.

A simplified 2D plane-parallel model of the transformer was developed to estimate the fluxes in the core in a short circuit operation mode. The main assumptions of this model are: (a) the magnetic permeability of the yoke is quite large; (b) the height of the transformer window is much bigger than the window width. The model gives an estimate for the ratio between the fluxes in the short circuit and the idling modes for the central core and side yokes (k_C and k_S respectively) [5]:

$$k_S^{(1)} \approx 1 + \frac{b}{2a + 6\delta + 2b}, \quad k_C^{(1)} \approx -\frac{a}{2a + 6\delta + 2b}. \qquad (2)$$

The inner coil here is assumed to be short circuited. For the transformer with the configuration shown in Fig. 1 and Table 1 these coefficients are equal:

$$k_S^{(1)} = 1.33, \quad k_C^{(2)} = -0.083.$$

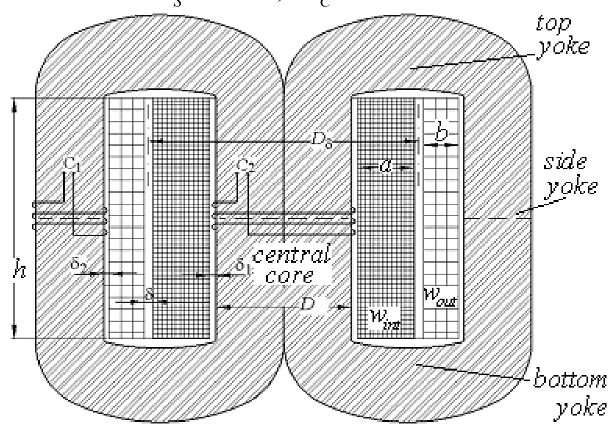

Fig. 1. Cross section of the transformer with probe coils C_1 and C_2.

To calculate the short circuit parameters it is enough to evaluate transformer inductances in the idling mode. The exact value of the relative permeability is not essential for considered problem if it exceeds the unit significantly. For the main calculations we have taken a constant permeability of $\mu_r = 5000$. The currents in the coils in a short-circuit mode may be presented by a relation:

$$I_1 \approx \frac{U}{2\pi f \cdot L_1}\cdot\frac{1}{1-k^2} \qquad I_2 \approx I_1\frac{M}{L_2}, \qquad (3)$$

where M is a mutual inductance, L_1 and L_2 are the inductances of the transformer winding, f is the applied voltage frequency. The resistances of the coil are neglected.

For the applied voltage frequency of 1 kHz this assumption is a good approach. The coupling coefficient between the windings $k = M/\sqrt{L_1 \cdot L_2}$ is very close to one unit for the considered transformer ($k^2 \approx 0.9998$). That is why the coil currents exceed significantly the nominal values.

TABLE I
PARAMETERS OF THE TRANSFORMER

Item	symbol	units	value
Central yoke width	D	mm	56
Central core – coil gap	δ_1	mm	3
Inner coil width	a	mm	3
Outer coil width	b	mm	12
Outer coil – side yoke gap	δ_2	mm	2
Gap between coils	δ	mm	1
Coil height	h	mm	84
Core length	l	mm	82
Inner coil turns number	w_{ext}		162
Outer coil turns number	w_{int}		648

III. Computation of the Magnetic Field and Flux Couplings in the Transformer

To calculate the magnetic field distribution in the ferromagnetic part of the transformer the integro-differential equation (1) was solved numerically for 1/8th part of the core. The full 3D model is shown in Fig.2. The first order approximation was used; the total number of tetrahedral elements was equal to 67584. These calculations were used to evaluate the inductances of the transformer windings.

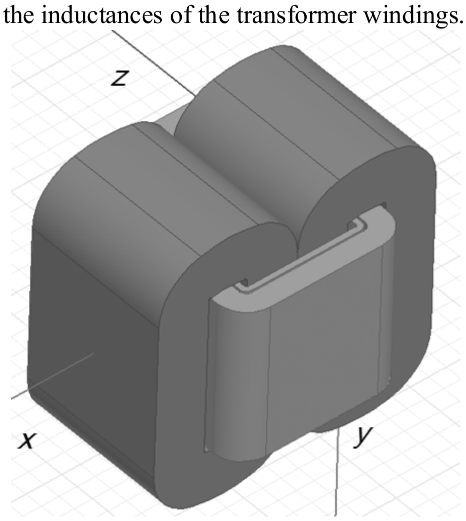

Fig. 2. A full 3D model of the transformer

A solution of the integro-differential equation gives a reasonably good field distribution inside the ferromagnetic part of the construction. But outside the core an accuracy of the field computation is not satisfactory. In the vicinity of the ferromagnetic parts the uniformly magnetized individual elements of the core distort strongly the real field distribution. This effect is shown in Fig.3. That is why a special post-processing procedure was applied. In the space around the ferromagnetic core the reduced magnetic potential was used for approximating field intensity. The boundary conditions at the borders of the selected volume were taken from the solution of the integro-differential equation if the corresponding part of the border coincided with the surface of the core or were evaluated using integral presentation of the potential. After defining the boundary conditions the reduced potential distribution is evaluated using finite element technique. The final field intensity is evaluated as a sum of the components induced by the winding and the magnetized core:

$$\vec{H} = \vec{H}_c - \vec{\nabla} U_m . \tag{4}$$

Such post-processing procedure was used before for the shielding systems where the boundary conditions are completely taken from the solution of the integro-differential equation [6]. We used this procedure to evaluate the flux couplings with the transformer coils and consequently the flux coefficients $k_S = 1.357$ and $k_C = -0.163$. The experimental investigation gave close values: $k_S^{(\exp)} = 1.33$, $k_C^{(\exp)} = -0.171$ [7]. It is necessary to note, that deviation of the computed inductances even by 10^{-4} rel. units destroyed completely the quality of the solution.

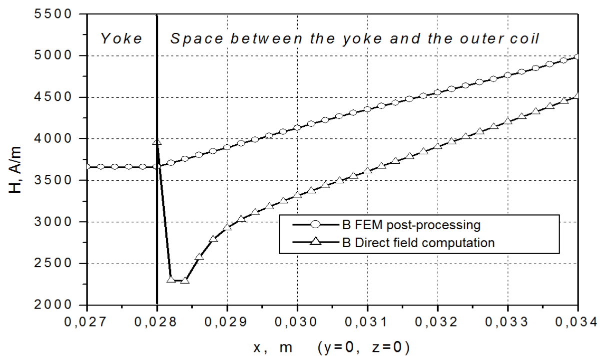

Fig. 3. Field intensity distribution near the yoke border.

IV. Conclusions

The integro-differential equation for the scalar magnetic potential has been used for modeling the magnetic field in the closed core of the electric transformer. The post-processing at the basis of finite element technique ensured a high precision of the computed field distributions. High accuracy of the calculated flux distributions in the transformer core was confirmed experimentally.

V. References

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